

Masters Exam, Algorithms: Fall 2000

Answer a total of four problems, two from the easy group, and two from the hard group.

Easy

1. Describe a situation in which heap sort would be faster than a relatively simple implementation of quicksort. Why do people usually use quicksort anyway?
2. Give a definition and draw a graph to illustrate each of the following.
 - $O(f(n))$
 - $\Theta(f(n))$
 - $\Omega(f(n))$
3. How many different orders can we place the letters A, B, C, D, and E in?
4. What is the complexity of a sorting algorithm that obtains a result by inserting each element at the correct location in a linked list?

Hard

1. Assume we have a directed graph with no cycles. Sketch pseudocode to find the longest path through this graph.
2. The Ford-Fulkerson algorithm may perform poorly on some graphs; the Edmonds-Karp algorithm fixes this problem. Show a graph where FF performs poorly, and explain what happens.
3. We have a set of tasks, and a set of employees. Each employee can handle a few tasks, but not necessarily all of them. Each employee can be assigned one and only one task, and each task requires one and only one employee. How would we determine an assignment of employees to tasks such that we handle the maximum number of tasks possible? It is sufficient to explain how to apply a well known algorithm to this problem.
4. Suppose we have a cookie making factory that makes square cookies. We first bake one single large cookie in a rectangular pan, and then make the individual cookies by cutting apart the large cookie. It costs us k^2 to make a slice k units long, and we have to slice all the way across the cookie with each cut. For example, the 3 by 2 cookie in the figure below can be sliced with one long slice and four short slices (total cost $3^2 + 1^2 + 1^2 + 1^2 + 1^2 = 13$) or with two medium length slices, and three short ones (total cost $2^2 + 2^2 + 1^2 + 1^2 + 1^2 = 11$). Describe an algorithm to determine the optimal (minimum cost) way to slice an m by n cookie, and explain why it's optimal.

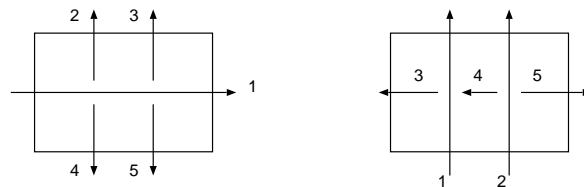


Figure 1: A 2 by 3 cookie can be sliced in different ways. The first way has a total cost of 13, while the second way has a total cost of 11.