

The first question is required; pick two questions from the “easy” group, two questions from the “medium” group, and one question from the “hard” group. Please put no more than one answer on a page, and indicate which question you are answering.

REQUIRED: Give formal definitions for O, Omega, Theta, P, and NP.

EASY QUESTIONS: PICK ANY TWO

EASY 1: Describe what is meant by “amortized analysis.” For example, amortized analysis is used when evaluating algorithms for disjoint sets.

EASY 2: What does “pseudopolynomial” mean when describing an algorithm. Give an example of such an algorithm.

EASY3: You are mapping an algorithm to a new multi-core microprocessor. Does the computational complexity of the algorithm change? If the microprocessor has four cores (all running at the same speed as an original single-core processor), can we determine anything about the change in run time?

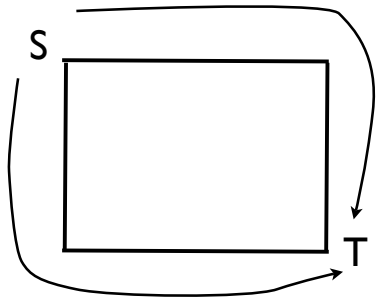
MEDIUM QUESTIONS: PICK ANY TWO

MEDIUM 1: Give an optimal prefix-free (Huffman) code for the following letters, where we have the indicated frequencies: 10 As, 5 Bs, 17 Cs, 3 Ds, 11 Es, 4Fs.

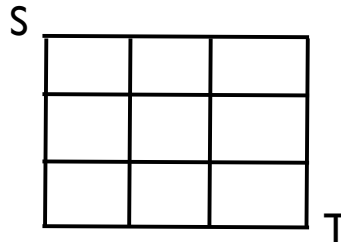
MEDIUM 2: Describe briefly the differences between Kruskal’s algorithm and Prim’s algorithm. You should provide a bit of pseudocode to describe them.

MEDIUM 3: Suppose you have a box that can carry a fixed maximum weight. You have a set of materials (for example, gold dust, sand) that each have a unique value per unit weight. You can take as much of these materials (but there may be only a limited amount of any of them). Describe an algorithm that would select an optimal set of materials to put into the box to maximize the total value. Provide a bit of pseudocode for this.

MEDIUM 4: Assume you are in a city with streets that go only north-south, and east west. You start at an intersection marked S, and want to travel to an intersection marked T. How many different ways are there to go from S to T if you only travel east and south (to the right and down)? The example on the left shows a simple example, where there are only two ways. Fill in the answer for the question on the right. Do not “brute force” this; try to be clever!



2 ways



_____ ways

HARD QUESTIONS: PICK ANY ONE

HARD 1: Describe the MAX-FLOW MIN-CUT theorem. There are three conditions that are true for a maximum flow in a network, and if any of the conditions are true, they all must be true. State what those conditions are.

HARD 2: You travel to a European city famous for its bridges; you decide to take a walk, starting at your hotel. You wish to cross every bridge once and only once, returning to your starting place at the end of the walk. Can you determine if such a walk is possible in general (you do not need to provide pseudocode for this)? If you can think of a related problem that has been well studied in computer science, you should include it in your answer.

The European city is also famous for street intersections. Is it possible to walk through the city, going through each intersection once (and only once)? Again, you do not need to provide pseudocode, and there might be a related problem that should be part of your answer.

HARD 3: What is meant by a “polynomial time reduction”? Describe where this might be used, in particular with reference to NP-Completeness.